MATH 483-1: PROBLEM SET 5

Due Wednesday, December 10

(1) [Ha] Ch.II: 1.18 (and also the adjoint property of $f^*$ and $f_*$ on page 110)

(2) [Ha] Ch.II: 5.1 (b), (c),(d), 5.4, 5.8, 5.17

(3) [Ha] Ch.II: 6.9

(4) Let $f : X \to Y$ be a dominant rational map of varieties. Assume that $X$ is smooth and that $Y$ is proper over the ground field. Show that $f$ pulls back global differential forms on $X$, i.e. that there is a natural pull-back map $f^* : \Omega_Y(Y) \to \Omega_X(X)$. (Hint: recall that by the valuative criterion of properness the rational map $f$ is actually a morphism outside a closed subset of codimension $\geq 2$.)

(5) [Ha] Ch.II: 8.3, 8.4. Some parts of these problems refer to the notion of arithmetic genus. We have not discussed this; if you’re not familiar with it, you can just skip them and wait until we discuss Hilbert polynomials later in the course.

Bonus problems: The exercises in Ch.II §5 in [Ha] contain many other theoretical facts that anyone seriously interested in the subject should know about; the most important ones are 5.9, 5.10, 5.14, 5.16, 5.18. I’ll be happy to look at them (and add the scores) if you wish to turn them in at any point.