Math B14-3 Calculus **Autumn 1998**

Quiz 5 Review

The topics for this quiz are:

1. Tangent planes (§13.4)
2. Critical points of functions of several variables (§13.5)
3. Second derivative test (§13.10)
4. Applied max-min problems (§13.5)

You should be familiar with all the assigned homework for these sections. Below are some sample problems. Any applied max-min problem would be limited in scope because of the time allowed. See no. 4 below.

1. Find the tangent plane to the surface $z = \sin(\pi xy)$ at the point $(1, 1, 0)$

2. Show that partial derivatives of $f(x, y) = \cos(\pi(x^2 + y^2))$ are zero when $x = 1$ and $y = 0$. Is this a maximum, a minimum, or neither?

3. Find and classify all critical points of

$$f(x, y) = xy + 3x - 2y + 4.$$

Use the second derivative test to verify your answers.

4. Find the point on plane $x + y + z = 1$ which is the minimum distance from the origin $(0, 0, 0)$.

Answers

1. $\pi x + \pi y + z = 2\pi$.

2. The partial derivatives are

$$\frac{\partial f}{\partial x} = -2\pi x \sin(\pi(x^2 + y^2)) \qquad \text{and} \qquad \frac{\partial f}{\partial y} = -2\pi y \sin(\pi(x^2 + y^2)).$$

These will be 0 whenever $x^2 + y^2 = 1$ (among other places). In particular, they are zero at $(1, 0)$, and this is a minimum since $f(x, y) \geq -1 = f(1, 0)$ for all (x, y). It is not an isolated max or min, however. (Draw the graph!)

3. The only critical point is $(2, -3)$. Here $\Delta = -1$, so this is a saddle point.

4. The distance from (x, y, z) to $(0, 0, 0)$ is $\sqrt{x^2 + y^2 + z^2}$. It is sufficient to minimize the sqaure of this distance. Also, $z = 1 - x - y$, so the function to be minimized is

$$f(x, y) = x^2 + y^2 + (1 - x - y)^2.$$

The minimum occurs at $x = y = z = 1/3$.