MATH 281-2

Test 2 Review

1. a.) A damped spring system is governed by

\[ y'' + \gamma y' + 5y = 0. \]

Any solution \( y \) passes through zero infinitely many times. What can you say about \( \gamma \)?

b.) For a different system, there are solutions of

\[ y'' + \gamma y' + 4y = \sin(2t). \]

which attain arbitrarily large values. What’s \( \gamma \)?

2. Find the solution of the initial value problem

\[ y'' - 2y' + y = 2e^t, \quad y(0) = 0 = y'(0). \]

3. Consider Bessel’s equation

\[ x^2y'' + xy' + (x^2 - 4)y = 0. \]

a.) Write down the indicial polynomial and find the largest root \( r_1 \).

b.) Write down the first three non-zero terms of the series solution (note \( a_0 = 1 \)).

\[ y(x) = x^{r_1}(1 + a_1 x + a_2 x^2 + \cdots). \]

c.) Use your answer to estimate \( y(0.1) \) with an error less than .0001. Be sure and explain how you estimated the error.

4. A tank of 200 liters initially contains pure water, but a solution of salt flows in at a rate of 1 l/min and mixed solution flows out at the same rate. Suppose the incoming solution has concentration 2 g/l for the first 60 minutes, 4g/l for the second 60 minutes, and is pure water after that.

a.) Write down the IVP for the total amount of salt in the tank at time \( t \).

b.) Solve this initial value problem.
Answers

1a.) The characteristic polynomial must have complex roots, so γ < 2√5.

b.) γ = 0.

2. y(t) = t²eᵗ.

3. a.) The indicial polynomial is r² - 4 and r₁ = 2.

b.) y(t) = x²(1 - \frac{1}{12}x² + \frac{1}{384}x⁴ - \cdots).

c.) Since this is an alternating series and the term \frac{1}{384}(0.5)^6 < .0001, we have

   y(0.5) = (0.5)^2(1 - \frac{1}{12}(0.5)^2) \sim 0.2448.

4. a.) y′ + \frac{1}{200}y = 2 + 2u_{60}(t) - 4u_{120}(t)

b.) y(t) = 400f(t) + 400u_{60}f(t - 60) - 800u_{120}f(t - 120) where

   f(t) = 1 - e^{-(1/200)t}.

Note that y(t) → 0 as t → ∞, as expected.