MATH 360-1, Fall 2009 **MENU Applied Analysis**

Project 2: Coupled Springs

This project is adapted from the document "Coupled Springs", a supplement to

Borelli, R.L. and Coleman, C.S, *Differential Equations: A Modeling Perspective*, John Wiley and Sons, Inc., New York 2004.

available on the class web site. The page numbers below will refer to the numbers in that document.

In class we examined a mathematical model for the motion of a mass attached to the end of single spring and we learned that the motion was always very regular, whether there was damping or not. In this project we add multiple masses and we will see that the motion is often much more irregular in nature. The basic assumptions remain the same.

Assumptions: Springs are subject to Hooke's law, any damping is proportional to velocity (viscous damping), and force is mass times acceleration.

1. The equation for two coupled springs is derived on pp. 59–60 of the supplement. However, the explanation is very short. Expand on this and explain how they got the equations. You will obtain a system of two 2nd order ODES; rewrite as four 1st order ODEs. (See Example 1.)

2. (Maple) Use Maple to recover the graphs of figures 1, 2, and 3 on pp, 61–62. Explain what the graphs mean. (You can do this as six separate graphs if you find this easier.) Speculate on why some solutions show very regular behavior, but others do not.

3. (Maple) Now reproduce Figure 4. Explain what this graph represents. Notice there is no time variable in this graph, so you may have to think for a moment about this. The curve (3) is by far the most likely outcome if you pick the initial conditions at random. You will notice that it remains inside a rectangle. What are the boundaries of the rectangle and what do they have to do with initial conditions?

5. Do problem 12 on p. 63. This asks you to find the general solution of the linear system (3) on p. 61. Which solutions are the *normal modes*? (The term is defined on p. 61.) More on normal modes can be found in the link on the class web page.

4. Do question 9 on p. 63. Don't neglect the damping. Why do you get an ODE almost identical to that of one mass and one spring?

5. Now do question 10. This again asks for damping.

6. (Maple) Do problem 11 without damping. My instructions here are short, but this problem is quite long and you will create a significant number of graphs. Be choosy in what you present in your final report; your examples should illustrate your points.

7. (Maple) Do problem 11, now with the damping listed there. Again graph x_1 and x_3 versus time and x_1 versus x_3. All solutions should die off, but explain what's happening as they do so.

8. (Maple) Do problem 13. Again this is a longer problem and you'll create multiple graphs. The end of the problem asks questions about amplitude and frequency. This can be done by a formula (the "Phase shift" formula on the inside back cover of your book), but you can estimate these from the graphs if you prefer.