Project 1

This project is adapted from Chapter 2.7 of our text and from Chapter 1.8 of

Borelli, R.L. and Coleman, C.S, *Differential Equations: A Modeling Perspective*, John Wiley and Sons, Inc., New York 1998.

The purpose of this project is to model the flow of medication through the human body. We will treat the various parts of the body as compartments and the track the medication as it enters and leaves each compartment. In this way, the mathematics will be very close to the lake pollution model. Two typical compartments are the gastrointestinal (GI) tract and the blood stream. These can be be broken down more finely (stomach versus liver, for example) and other compartments can be added. Here is our basic assumption:

Assumption: Medication leaves one compartment and enters another at a rate proportional the amount present in the first compartment.

The actual constant of proportionality k is determined experimentally, and is determined both by the medicine and the individual.

1. Assume an instantaneous does of A units of decongestant is introduced into the GI tract at time 0. From there is passes into the blood stream, where it actually goes to work. If

$$\begin{aligned} x(t) &= \text{amount of drug in GI tract} \\ y(t) &= \text{amount of drug in blood} \end{aligned}$$

write out the pair of differential equations for the determining x and y. Your answer should involve a constant k_1 for the GI tract and k_2 for the blood stream.

2. Solve these differential equations. Assume $k_1 \neq k_2$.

3. (Maple) Assume that rate constant for a certain cold pill are

$$k_1 = 0.6931 \text{ per hour} \qquad k_2 = 0.0231 \text{ per hour}$$

and $A = 1$. Use Maple to graph x and y. Estimate the highest level of antihistamine in the blood and when it occurs.

4. (Maple) Change the values to k_2 (the clearance coefficient from the blood stream) to the following values and graph these as well and discuss what you see.

$$\begin{aligned} k_2 &= 0.00231 \\ &= 0.00731 \\ &= 0.0231 \\ &= 0.0731 \\ &= 0.231 \end{aligned}$$

5. (Maple). We now vary k_1. Display the effects of antihistamine levels in the bloodstream for $k_1 = 0.06931$, 0.11, 0.3 ,0.691, 1.0, and 1.5. Suppose the desired range for antihistamine levels is between 0.2 and 0.8. Find the upper and lower levels of k_1 so that the levels in the blood reach 0.2 within 2 hours and stay below 0.8 for 24 hours.

6. Most medication is taken in repeated doses. We suppose that 6 units of antihistamine are delivered to the GI tract a constant rate over a half-hour span and then repeated every six hours. Assume there is none present to begin. Write out the differential equations for this new situation. Display initial conditions, but don't try to solve the equation. You will need the square wave function. See below for a primer on this function.

7. (Maple) Again assuming k_1 and k_2 as in Problem 3, graph the amount of antihistamine in the blood over 120 hours. You should see a disturbing tendency towards overdose. Specifically, suppose that if the amount of antihistamine rises above 30 units, the patient is groggy and drowsy. Estimate when this occurs. Now assume that if it goes above forty, the patient will fall asleep. Does this happen?

8. (Maple) Adjust the dosage level so that units in the bloodstream will rise above 15, but stay below 30 for 120 hours.

9. (Maple) The constants k_1 and k_2 depend heavily on the individual. For a young child, for example, they may be only fractions of that of a healthy adult. Assume that coefficients for a child are only (1/4) of that of an adult, again using the numbers of problems 3; also assume the safe zone is now 10 to 25 units in the blood. Experiment with dosage to achieve a safe and effective level in the blood. You can vary the level, but also the frequency of dosage.

The square wave function. This function is a periodic function which takes the value 1 for some percentage of the time and the value zero for the rest. It is written

$$\text{sqw}(t, \text{percentage}, \text{period})$$

For example

$$\text{sqw}(t, 50, 4)$$

takes the value 1 between 0 and 2 and zero between 2 and 4, then repeats and the function

$$25\ \text{sqw}(t, 75, 8)$$

takes the value 25 between 0 and 6 and zero between 6 and 8, then repeats. In question no. 6, you need the square wave function

$$12\ \text{sqw}(t, 100/12, 6) = 12\ \text{sqw}(t, 25/3, 6).$$

This has period of 6 hours, is on for 1/12 of the time (a half-hour) and delivers 6 units of medicine (the area under the curve) in each period.