MATH 360-1, Fall 2008 **MENU Applied Analysis**

Project 2

This project is adapted from Chapters 5.6 and 6.4 of our text and from the problems at the end of Chapter 5.5 of

Borelli, R.L. and Coleman, C.S, *Differential Equations: A Modeling Perspective*, John Wiley and Sons, Inc., New York 1998.

We will model the course of an epidemic through a large population. It is similar to, but not identical to, a predator-prey scenario, with the healthy population as prey and the contagious population as the predator.

Assumptions: We have some mild disease which is highly contagious, rarely fatal, can only be caught once, and runs its course in a short period. We will divide the population into three parts:

S = *Susceptibles*: those can catch the disease
C = *Contagious*: the sick who can transmit the disease
R = *Recovered*

We are assuming that all sick are contagious and that the duration of the disease is short enough that natural births and deaths don't change the population much.

1. Argue that changes in population over the course of the disease can be modelled by

$$S' = -bSC$$
$$C' = bSC - rC$$
$$R' = rC$$

for positive constants b and r and subject to the initial populations S_0 and C_0. Assume $R_0 = 0$. Also notice that $S + C + R$ is the entire population, hence constant. Why does this mean the equation for R' is redundant?

2. The value b depends not only on the disease, but on the environment in which the populations lives. Why? The value of r depends only on the disease and can be estimated in terms of the average time an individual is infectious. Why? (See p. 142-3 for an explanation and examples.)

3. Show that the disease spreads only if S_0 is large enough; that is, show that there is a value of S above which more people are infected (contagious) than recover. Also, what are the equilibrium solutions? Interpret your answer.

4. Suppose some disease lasts four days, that a susceptible meets 0.3% of the population each day, that the disease is transmitted in (1/6)th of the contacts. Find the values of b and r.

The remainder of the problems ask you to create some graphs using Maple. Be sure you provide some narration to analyze these graphs. What are you seeing? What does this say about the population? Does everyone get the disease? Is it happening quickly? Are the values stabilizing?

5. (Maple) Continuing with the same disease as in no. 4: what is the threshold level of problem 3? Plot the component graph of $C(t)$ for various choices of C_0 and various values of S_0 around this threshold level to see the growth and decline of C. Plot for 50 days. For which values do most of the population get ill?

6. (Maple) For another illness, suppose $b = 0.001$ and $r = 0.08$ and $C_0 = 100$ infected people get on the cruise ship and join the population. Investigate how the spread of infection depends on the size of the population by plotting S, C, and R graphs versus time for fifty days. Let S_0 range from 250 to 2000 in increments of 250. How does the value of S_0 affect the speed with which the disease runs its course?

7. (Maple) For the disease of problem 6, find C as function of S, C_0 and S_0. Plot C versus S for various values of S_0 and C_0, $0 < S_0 < 1600$, $0 < C_0 < 1250$. Include some phase planes. Note that

$$\frac{dC}{dS} = -1 + \frac{r}{bS}.$$

8. (Maple) Do problem no. 9, p. 154.

9. (Maple) In problem no. 13, p. 193, you will find the model for a disease with reinfection. (There are no recovereds anymore.) Return to the disease of problem no. 6 and again graph S and C with S_0 ranging from 250 to 2000 in increments of 250 – now assuming reinfection. Interpret what you see. You may have to run the time for 100 days to see stability.