MATH 360-1, Fall 2008 **MENU Applied Analysis**

Test 1

Each question is worth 20 points. Please show all your work.

1. A falling 3 kg object meets air resistance equal to twice its velocity. Assume it is falling straight down and the only other force acting is gravity.

a.) Write a differential equation to describe the the velocity of the object at time t. Assume force is mass times acceleration and write the gravitational constant as g.
b.) Assume the object is dropped, so $v(0) = 0$. Find the solution of differential equation.

2. Consider a system of two tanks, the first holding a 200l of salt solution, the second holding 100l of salt solution. A solution of 4 g/l of salt solution flows into the first tank at a rate of 2 l/min; thoroughly mixed solution flows to the second tank at a rate of 4 l/min. For the second tank, 3 l/min of 2 g/l of salt solution flows in, 2 l/min of solution flows out to the first tank and 5 l/min flows away.
a.) Write a system of differential equations for the amount of salt in the two tanks.
b.) Find the equilibrium solution of the system in part (a).

3. Suppose $\mathbf{z}' = \begin{pmatrix} a & -1 \\ 1 & a \end{pmatrix} \mathbf{z}$.
a.) Give a condition on a so that all solutions go to 0 as $t \to \infty$.
b.) Solve the equation with $a = -1$ subject to $\mathbf{z}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

4. In this problem we will study a population y subject to the harvested logistic equation

$$y' = 6y - y^2 - 8.$$

a.) Draw the phase portrait, indicating the equilibrium solutions and where the population will be rising or falling.
b.) Suppose $y(0) = 1$. Use the phase portrait to predict whether the population survives.
c.) Solve the differential equation subject to $y(0) = 1$.

Extra credit: Your answer to 4.c.) should show $y \to 4$ as $t \to \infty$. Why does this not contradict your answer to 4.b.)?

Answers

1. If down is positive, $v' = g - (2/3)v$ where g is the gravitational constant.

b.) $v = \dfrac{3g}{2}(1 - e^{-(2/3)t})$.

2.a.) $\mathbf{z}' = \begin{pmatrix} -\frac{1}{50} & \frac{1}{50} \\ \frac{1}{50} & -\frac{7}{100} \end{pmatrix} \mathbf{z} + \begin{pmatrix} 8 \\ 6 \end{pmatrix}$

b.) $\mathbf{z} = \begin{pmatrix} 680 \\ 280 \end{pmatrix}$.

3.a.) We need $a < 0$ as the eigenvalues are $a \pm i$.

b.) $\mathbf{z} = \begin{pmatrix} e^{-t}\cos t \\ e^{-t}\sin t \end{pmatrix}$.

4.a.) The equilibrium solutions are $y = 2, 4$, with y rising for $2 < y < 4$ and falling otherwise.

b.) The population goes to zero.

c.) $y = \dfrac{4 - 6e^{-2t}}{1 - 3e^{-2t}}$

d.) The graph of y has a vertical asymptote when the denominator is zero; that is, at $t = (1/2)\ln(3)$. (The population is zero at $(1/2)\ln(3/2)$.) For this reason, the long-term behavior of y is irrelevant to the original initial value problem.