MATH 360-1, Fall 2008 **MENU Applied Analysis**

Practice Test 1

1. Newton's Law of Cooling says that the rate an object cools is proportional to the difference between the temperature of the object and the ambient temperature.

a.) Write a differential equation to describe the temperature of such an object as a function of time. Be sure and specify the initial condition.

b.) Show that your differential equation predicts that the equilibrium solution is the ambient temperature.

c.) Coffee at 90 degrees C is brought into a 20 degree room. After ten the coffee has cooled to 40 degrees. At what time will the coffee be at 22 degrees?

2. In this problem we will study a population subject to the logistic equation

$$y' = y(6 - y).$$

a.) Draw the phase portrait, indicating the equilibrium solutions and where the population will be rising or falling.

b.) Suppose $y(0) = 3$. Use the phase portrait to predict the value of y at $t \to \infty$.

c.) Solve the differential equation subject to $y(0) = 3$.

3. Consider a system of two tanks, the first holding a 100l of salt solution, the second holding 200l of salt solution. A solution of 2 g/l of salt solution flows into the first tank at a rate of 1 l/min; thoroughly mixed solution flows to the second tank at a rate of 2 l/min. From the second tank, 1 l/min of solution flows into the first tank and 1 l/min flows away.

a.) Write a system of differential equations for the amount of salt in the two tanks.

b.) Find the equilibrium solution of the system in part (a). Is this a valid answer?

4. Consider the system of ODEs

$$\mathbf{z}' = \begin{pmatrix} a & 2 \\ 2 & a \end{pmatrix} \mathbf{z}.$$

a.) Find a condition on a so that all solutions will tend to zero at $t \to \infty$.

b.) Find the general solution if $a = -3$.

Over for Answers

Answers

1.a.) $T' = -k(T - T_a)$ with T_0 the temperature at time 0. Here T_a is the ambient temperature.

b.) If $T' = 0$, then $T = T_a$.

c.) $T = 20 + 70e^{-kt}$ with $k = (1/10)\ln(7/2) = 0.125$. So $T = 22$ when $t = 28.4$ minutes.

2. a) The equilibrium solutions are $y = 0, 6$. The population is rising if $0 < y < 6$ and falling if $y > 6$.

b,) $y \to 6$.

c.) $y = 6e^{6t}/(1 + e^{6t}) = 6/(1 + e^{-6t})$.

3.a.) $\mathbf{z}' = \begin{pmatrix} -\frac{1}{50} & \frac{1}{200} \\ \frac{1}{50} & -\frac{1}{100} \end{pmatrix} \mathbf{z} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

b.) $\mathbf{z} = \begin{pmatrix} 200 \\ 400 \end{pmatrix}$ which is 2 g/l – the incoming solution – throughout the system.

4.a.) The eigenvalues are $a + 2$ and $a - 2$. Thus we need $a < -2$.

b.) $\mathbf{z} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-5t}$.