MATH 360-1, Fall 2008 **MENU Applied Analysis**

Practice Test 2

1. A damped spring system is governed by

$$y'' + \gamma y' + 5y = 0.$$

a.) We know that any non-zero solution passes through $y = 0$ infinitely many times. What does this say about γ?

b.) Now suppose you know that $y(t) = 0$ only when t is a multiple of $\pi/2$. What's γ?

c.) For a different system, there are solutions of

$$y'' + \gamma y' + 4y = \sin(2t).$$

which attain arbitrarily large values. What's γ?

2. Consider the system of equations

$$x' = -xy$$
$$y' = -2y$$

where differentiation is with respect to time. Find an expression for dy/dx and write y as a function of x given the initial conditions $x(0) = 1$ and $y(0) = 4$.

3. The following system of differential equations governs two competing species.

$$x' = x(1 - x - y)$$
$$y' = y(0.75 - y - 0.5x)$$

a.) Draw and analyze the complete phase plane, showing where $x' = 0$, where $y' = 0$ and the behavior of the direction fields in various parts of the first quadrant. You should find and clearly label four different such areas.

b.) What are the equilibrium solutions? (There are four.) Decide if they are stable (attractors) or not.

4. A cylindrical piston, with moving end up, is filled with a gas subject to the ideal gas law

$$PV = nRT.$$

Assume the only forces acting on the moving part of the piston are gravity and the pressure P of the gas. Write the mass of the moving end of the piston as m, the radius of the piston R; further assume that the amount of gas present (the quantity nR) and the temperature T are constant, so that only the volume V and pressure P vary.

a) Write a differential equation describing the motion of the moving part of the piston.

b) Show that the equilibrium solution is a center.

Answers

1a.) $\gamma < 2\sqrt{5}$.
b.) $\gamma = 4$.
c.) $\gamma = 0$.

2. $y = 2\ln(x) + 4$.

3b.) The four equilibria are $(0,0)$, $(1,0)$, $(0,3/4)$ and $(1/2,1/2)$. The first is a repeller, the second two are saddle points, and the last is an attractor.

4. a.) Let y be the height of the interior of the piston, so that $V = \pi r^2 y$. Then

$$my'' = \frac{nRT}{\pi r^2 y} - mg$$

or

$$y'' = \frac{K}{y} - g$$

where K is a constant. The equilibrium solution is at $y_e = K/g$.
b.) Rewrite the differential equation as

$$\begin{aligned} u' &= v \\ v' &= \frac{K}{u} - g \end{aligned}$$

with $u = y$ and $v = y'$. The Jacobian of this system at $u = y_e$ and $v = 0$ is

$$J(y_e, 0) = \begin{pmatrix} 0 & 1 \\ -K/y_e^2 & 0 \end{pmatrix}$$

which has purely imaginary roots.