Math 291-3: Final Exam
Northwestern University, Spring 2018

Name: ______________________________

1. (15 points) Determine whether each of the following statements is true or false. If it is true, explain why; if it is false, give a counterexample.

(a) If $f : [-4,5] \times [1,3] \to \mathbb{R}$ is continuous, then f is integrable over $[-4,5] \times [1,3]$.

(b) The sum

$$\int_{-1}^{0} \int_{-x}^{\sqrt{2-x^2}} x^2 y \, dy \, dx + \int_{0}^{1} \int_{x}^{\sqrt{2-x^2}} x^2 y \, dy \, dx$$

can be written as a single iterated integral in polar coordinates.

(c) The value of the line integral $\oint_{\partial D} -y \, dx + (x^2 + x) \, dy$ depends only the area of D.

Problem	Score
1	
2	
3	
4	
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7	
Total	

2. (10 points) For a continuous function $f : \mathbb{R}^3 \to \mathbb{R}$, consider the following iterated integral.

$$\int_0^{\pi/2} \int_0^1 \int_r^{\sqrt{2-r^2}} f(r\cos\theta, r\sin\theta, z)\, r\, dz\, dr\, d\theta$$

(a) Rewrite this as a **sum** of iterated integrals in rectangular coordinates with respect to the order $dy\, dz\, dx$.

(b) Rewrite this as a **single** iterated integral in spherical coordinates.

3. (10 points) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying

$$\int_0^1 (1-u)f(u)\,du = 10.$$

Find the value of the following double integral.

$$\int_0^1 \int_0^x f(x-y)\,dy\,dx$$

Hint: Make a change of variables with $u = x - y$ and v being something I'll leave for you to determine.

4. (10 points) Let $\mathbf{F}$ be a C^1 vector field on $\mathbb{R}^n$ and suppose $\mathbf{x}(t), a \leq t \leq b$ is a parametrization of a flow line C of $\mathbf{F}$. Show that $\int_C \mathbf{F} \cdot d\mathbf{s} > 0$.

5. (10 points) Determine the value of the following line integral, where C is the left half of the circle $(x-1)^2 + y^2 = 1$ oriented clockwise from $(1, -1)$ to $(1, 1)$.

$$\int_C (e^y + y^2 + 1)\, dx + (xe^y + 2xy + \cos y + x)\, dy$$

Hint: Find a way to use the Fundamental Theorem of Line Integrals.

6. (10 points) Suppose S is a smooth C^1 surface in $\mathbb{R}^3$ without boundary and that $\mathbf{F}$ is a C^2 vector field on S. Show that $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = 0$. Careful: $\operatorname{curl} \mathbf{F}$ is not necessarily C^1 over the entire solid enclosed by S, so Gauss's Theorem may not apply.

7. (10 points) Let S be the portion of the cone $y = \sqrt{x^2 + z^2}$ between $y = 0$ and $y = 1$, oriented with normal vectors pointing away from the y-axis. Determine the value of the following surface integral.

$$\iint_S \left\langle x - y^{\sin z + 5}, y^2 - y, -yz - e^{\cos x + 100} \right\rangle \cdot d\mathbf{S}$$

Hint: At some point, using "cylindrical coordinates" adapted to the xz-plane might be helpful.