Math 300: Midterm 1
Northwestern University, Winter 2019

Name: ______________________________

1. Give an example of each of the following with brief justification.

(a) (5 points) An upper bound of $A := \left\{x \in \mathbb{R} \;\middle|\; x^3 \leq 3\right\}$ which is not the supremum of A.

(b) (5 points) A subset A of $\mathbb{R}$ and a subset B of $\mathbb{R}^2$ such that $\mathbb{R}^2 - (A \times A) \neq B$.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Let $n \in \mathbb{Z}$. Show that 5 divides $6n$ if and only if 5 divides n. You may use basic properties of even and odd integers (i.e. what happens when you multiply two odd integers together, two even integers together, or an odd with an even), but no other properties of relatively prime integers. For instance, saying something along the lines of "if $6n$ is divisible by 5, then n is divisible by 5 since 6 and 5 are relatively prime" is not enough.

3. (10 points) Let A and B be sets. Show that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

4. At no point in either part below can you take some kind of limit. You must find a different way.

(a) (5 points) Show that if x satisfies $6 - r^2 < x$ for all $r > 0$, then $6 \leq x$.

(b) (5 points) Determine the following intersection and prove that your answer is correct.

$$\bigcap_{r>0} (6 - r^2, 9)$$

To be clear, $(6 - r^2, 9)$ denotes the interval $\{x \in \mathbb{R} \mid 6 - r^2 < x < 9\}$ and the intersection is taken as r ranges among all positive real numbers.

5. (10 points) Let $A := \left\{ 3 - \frac{2}{\sqrt{n}} \;\middle|\; n \in \mathbb{N} \right\}$. Determine, with proof, the supremum of A. It would probably be easiest to use the fact that an upper bound b of A is the supremum of A if and only if for all $\epsilon > 0$, there exists $x \in A$ such that $b - \epsilon < x$, which you can use without justification.