Math 320-3: Midterm 2
Northwestern University, Spring 2015

Name: _____

1. (10 points) Give an example of each of the following. No justification is required.
(a) A bounded subset of $\mathbb{R}^3$ which is not a Jordan region.
(b) A bounded function on $[0,1] \times [0,1]$ which is not integrable.
(c) A non-constant function on $[0,1] \times [0,1]$ whose iterated integrals exist and are equal.
(d) A function $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ which sends the rectangle $[0,1] \times [0,2\pi]$ to the ellipse $x^2 + 2y^2 \leq 1$.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) For a Jordan region E of $\mathbb{R}^2$, let $(1,1)+E$ denote the set obtained by adding $(1,1)$ to each point of E:

$$(1,1)+E := \{(1+x, 1+y) \mid (x,y) \in E\}.$$

Show that $(1,1)+E$ is also a Jordan region.

3. (10 points) Define $f : [0,1] \times [0,1] \to \mathbb{R}$ by

$$f(x,y) = \begin{cases} x & \text{if } y = \frac{1}{n} \text{ for some } n \in \mathbb{N} \\ 0 & \text{otherwise.} \end{cases}$$

Show that f is integrable over $[0,1] \times [0,1]$ and determine the value of its integral.

4. (10 points) Suppose that A and B are two Jordan regions in $\mathbb{R}^2$ such that for any vertical line L, the intersection of A with L has the same length as the intersection of B with L. Show that A and B have the same area.

5. (10 points) Show that for any strictly positive continuous function $f : \mathbb{R}^2 \to \mathbb{R}$, we have

$$\int_{B_2(1,1)} 2f(x,y)\, d(x,y) > \int_{B_1(0,0)} 6f(1+2u, 1+2v)\, d(u,v).$$

To be clear, $B_2(1,1)$ denotes the disk of radius 2 centered at $(1,1)$ and $B_1(0,0)$ the disk of radius 1 centered at $(0,0)$.