These problems are for your own benefit, please do not submit solutions for grading.

1) Let $U \subset \mathbb{R}^2$ be open and connected, and let $\varphi : U \to \mathbb{R}^3$ be a global parametrization of a surface $S$. Suppose that $\frac{\partial \varphi}{\partial u}$ is independent of $u$ and $v$. Show that the Gaussian curvature of $S$ is identically zero.

2) Let $U \subset \mathbb{R}^2$ be the unit disc and let $\varphi : U \to \mathbb{R}^3$ be given by $\varphi(u, v) = (u, v, 0)$. Call $R$ the image of $\varphi$, which is a regular region in the $xy$-plane. Verify directly the Gauss-Bonnet formula for $R$ by computing

$$\int_R K dA + \int_{\partial R} \kappa_g ds,$$

and showing that the result matches with what is predicted by Gauss-Bonnet.

3) Let $S$ be the unit sphere in $\mathbb{R}^3$ and let $R$ be the simple region in $S$ given by

$$R = \{(x, y, z) \in S \mid x, y, z \geq 0\}.$$

Verify directly the Gauss-Bonnet formula for $R$ by computing

$$\int_R K dA + \int_{\partial R} \kappa_g ds + \sum_i \alpha_i,$$

and showing that the result matches with what is predicted by Gauss-Bonnet.