These problems are for your own benefit, to help you prepare for the final. You should also review all the past quizzes, midterms, and extra exercises on the course website.

1) Let \( \gamma \) be a smooth regular curve on the unit sphere \( S \). Show that if curvature of \( \gamma \) is always equal to 1 then \( \gamma \) is a geodesic.

2) Let \( \varphi : U \to \mathbb{R}^3 \) be a global parametrization of a surface \( S \), with unit normal vector \( N \). Given a smooth function \( h : U \to \mathbb{R} \) let

\[
\tilde{\varphi}(u, v) = \varphi(u, v) + h(u, v)N(\varphi(u, v)).
\]

Assume that \( \tilde{\varphi} \) gives a parametrization of a surface \( \tilde{S} \). Show that the first fundamental form of \( \tilde{S} \) satisfies

\[
\tilde{E} \geq E - 2he, \quad \tilde{G} \geq G - 2hg,
\]

where \( E, F, G \) and \( e, f, g \) are the coefficients of the first and second fundamental form of \( S \).

3) Let \( S \) be the surface of the cylinder given by \( x^2 + y^2 = 1 \) in \( \mathbb{R}^3 \), with the outward unit normal, and let \( R \) be the regular region inside \( S \) defined by \( 0 \leq z \leq 2 + x \). The boundary of \( R \) consists of two simple closed curves, \( C_1 \), the top one, and \( C_2 \), the bottom one. Using the Gauss-Bonnet formula for \( R \), compute \( \int_{C_1} \kappa_g(s)ds \).

4) Let \( S \) be the surface of a torus, and \( R \) be the regular region obtained by removing a small disc from \( S \). By drawing a triangulation of the region \( R \), compute its Euler characteristic \( \chi(R) \).

Hint: to represent the torus you should use the diagram used in class, with a square where the two pairs of opposite sides are identified.