1) Let \( B, B' \) be two path connected, locally path connected and semilocally simply connected spaces, and \( p : E \rightarrow B, p' : E' \rightarrow B' \) be their universal covers.

(a) For every continuous map \( f : B \rightarrow B' \) show that there is a continuous map \( \tilde{f} : E \rightarrow E' \) such that \( p' \circ \tilde{f} = f \circ p \).

(b) What is a necessary and sufficient condition for \( \tilde{f} \) to be unique?

(c) Fix a basepoint \( b \in B \), and assume that \( f \) is injective. Show that \( \tilde{f} \) is injective if and only if \( f_* : \pi_1(B, b) \rightarrow \pi_1(B', f(b)) \) is injective.

2) Let \( X \) be the topological space obtained from a (filled in) rectangle by identifying pairs of sides as shown:

\[
\begin{array}{c}
\text{a} \\
\downarrow \\
\text{b} \\
\end{array}
\quad \begin{array}{c}
\text{a} \\
\downarrow \\
\text{b} \\
\end{array}
\]

Calculate its fundamental group \( \pi_1(X, x_0) \) (relative to any base point \( x_0 \in X \)).

3) Let \( X = (S^1 \times S^1) \setminus \{p_1, \ldots, p_n\} \), where the \( p_j \)'s are distinct points in the torus \( S^1 \times S^1 \), and \( n \geq 1 \). Compute \( \pi_1(X, x_0) \) (relative to any base point \( x_0 \in X \)).

4) Let \( X = S^1 \times S^1 \) be the 2-torus, which we embed in \( \mathbb{C}^2 \) as usual by

\[
S^1 \times S^1 = \{(z, w) \in \mathbb{C}^2 \mid |z| = 1 = |w| \}.
\]

Let \( f : S^1 \times S^1 \rightarrow S^1 \times S^1 \) be the covering space given by

\[
f(z, w) = (z^aw^b, z^cw^d),
\]

for some \( a, b, c, d \in \mathbb{Z} \) with \( ad - bc \neq 0 \) (you don’t need to prove that it is a covering space). Compute the number of sheets of \( f \).

Hint: use the universal cover \( \mathbb{R}^2 \rightarrow S^1 \times S^1 \), and first show that the lifted map \( \tilde{f} : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) (as in problem 1) can be chosen to be \( \tilde{f}(x, y) = (ax+by, cx+dy) \).