1) Let $\Lambda, \Lambda' \subset \mathbb{C}$ be two lattices, and $X = \mathbb{C}/\Lambda, X' = \mathbb{C}/\Lambda'$ be the corresponding complex tori. Let $F : X \to X'$ be a holomorphic map. Show that $F$ lifts to a holomorphic map $\tilde{F} : \mathbb{C} \to \mathbb{C}$ of the form $\tilde{F}(z) = \alpha z + \beta$ for some $\alpha, \beta \in \mathbb{C}$ with $\alpha \Lambda \subset \Lambda'$.

2) Show that $F$ as in problem 1) is a biholomorphism if and only if $\alpha \Lambda = \Lambda'$. Deduce that every 1-dimensional complex torus is biholomorphic to $\mathbb{C}/(\mathbb{Z} \oplus \tau \mathbb{Z})$ for some $\tau \in \mathbb{C}$ with $\text{Im} \tau > 0$.

3) Prove that for $\tau, \tau'$ in the upper half plane, the two complex tori $X = \mathbb{C}/(\mathbb{Z} \oplus \tau \mathbb{Z})$ and $X' = \mathbb{C}/(\mathbb{Z} \oplus \tau' \mathbb{Z})$ are biholomorphic if and only if

$$\tau' = \frac{a \tau + b}{c \tau + d},$$

for $a, b, c, d \in \mathbb{Z}$ with $ad - bc = 1$.

4) Let $X = \mathbb{C}/\Lambda$ with $\Lambda = \mathbb{Z} \oplus \tau \mathbb{Z}$, $\text{Im} \tau > 0$, and let $0 \in X$ be the image of the origin. Let $F : X \to X$ be a biholomorphism, which satisfies $F(0) = 0$. Show that $F$ is induced by the map $\tilde{F} : \mathbb{C} \to \mathbb{C}$ given by $\tilde{F}(z) = \alpha z$ and that one of the following must hold:

- $\alpha$ is a 4th root of unity, and (up to the equivalence explained in problem 3) $\Lambda$ is the square lattice (i.e. we can take $\tau = i$)

- $\alpha$ is a 6th root of unity, and (up to the equivalence explained in problem 3) $\Lambda$ is the hexagonal lattice (i.e. we can take $\tau = e^{2\pi i/6}$)

- if none of the two previous cases arises, then $\alpha = \pm 1$