1) Let \((X^n, g)\) be a Kähler manifold of complex dimension \(n > 1\), and let \(f\) be a smooth positive nonconstant real function on \(X\). Prove that the conformally rescaled Hermitian metric \(f \cdot g\) is never Kähler.

2) Let \(X = (\mathbb{C}^2 \setminus \{(0, 0)\})/\mathbb{Z}\), be the Hopf surface, where the \(\mathbb{Z}\)-action is generated by the dilation \(\lambda : (z, w) \mapsto (2z, 2w)\). By direct calculation, prove that
\[
H^{1,0}_\sigma(X) = 0, \quad H^{2,0}_\sigma(X) = 0.
\]
Then use Homework 2, problem 3, to show that \(H^{0,1}_\sigma(X) \neq 0\). This gives an example of a compact complex surface with \(H^{1,0}_\sigma(X) \not\cong H^{0,1}_\sigma(X)\).

3) Let \((X^n, g)\) be a Hermitian manifold, and as usual let \(\omega\) be the fundamental \((1, 1)\)-form of \(g\). Let \(\beta\) be a real \((1, 1)\)-form on \(X\) such that
\[
\omega^{n-1} \wedge \beta = 0.
\]
Prove that
\[
\ast \beta = -\frac{1}{(n-2)!} \beta \wedge \omega^{n-2},
\]
where \(\ast\) is the Hodge star operator of \(g\) (which is determined by \(\eta \wedge \ast \psi = \langle \eta, \psi \rangle_g \omega^n\) for all \((1, 1)\)-forms \(\eta, \psi\) on \(X\), where \(\langle \cdot, \cdot \rangle_g\) is the pointwise tensor inner product defined by \(g\)).