1) Calculate the de Rham cohomology groups of the $n$-dimensional torus $T^n = S^1 \times \cdots \times S^1$ ($n$ times).

2) Let $M, N$ be two manifolds with finite dimensional cohomology. Prove that

$$\chi(M \times N) = \chi(M)\chi(N),$$

where $\chi(M) = \sum_{i \geq 0} (-1)^i \dim H^i(M)$ is the Euler characteristic.

3) Let $G$ be a (not necessarily connected) compact Lie group with $\dim G > 0$. Show that $\chi(G) = 0$.

4) Let $M$ be a compact orientable $n$-manifold, with $n$ odd. Show that $\chi(M) = 0$. 