1) Let $\mathcal{D}$ be the distribution on $\mathbb{R}^3$ spanned by the vector fields

$$X = \frac{\partial}{\partial x} + yz \frac{\partial}{\partial z}, \quad Y = \frac{\partial}{\partial y},$$

(a) Write down an integral submanifold for $\mathcal{D}$ passing through the origin

(b) Is $\mathcal{D}$ integrable?

(c) Can you write down an integral submanifold for $\mathcal{D}$ passing through $(0,0,1)$?

Let now $U(n) = \{A \in \operatorname{Mat}(n, \mathbb{C}) \mid A^* A = \operatorname{Id}\}$ be the unitary group, where $A^* = \overline{A^t}$, and $\operatorname{Mat}(n, \mathbb{C})$ is the space of $n \times n$ matrices with complex entries, and let

$$SU(n) = \{A \in \operatorname{Mat}(n, \mathbb{C}) \mid A^* A = \operatorname{Id} \text{ and } \det A = 1\}$$

be the special unitary group.

2) Show that $U(n)$ and $SU(n)$ are Lie groups and compute their dimensions.

3) Show that $U(1)$ is diffeomorphic to the circle $S^1$.

4) Show that $SU(2)$ is diffeomorphic to the 3-sphere $S^3$. 