1) Let \((X, \omega)\) be a compact Kähler manifold of complex dimension \(n > 1\), \(F : X \to \mathbb{R}\) be a smooth function with \(\int_X e^F \omega^n = \int_X \omega^n\) and let \(\varphi : X \to \mathbb{R}\) be a smooth function with \(\omega + \sqrt{-1} \partial \bar{\partial} \varphi > 0\) solving the complex Monge-Ampère equation
\[
(\omega + \sqrt{-1} \partial \bar{\partial} \varphi)^n = e^{F - \lambda \varphi} \omega^n,
\]
where \(\lambda\) is \(-1, 0\) or \(1\). Call \(g\) the Hermitian metric defined by \(\omega\) and \(\tilde{g}\) the one of \(\omega + \sqrt{-1} \partial \bar{\partial} \varphi\). In class we proved an identity for \(\Delta_{\tilde{g}} \text{tr}_{\tilde{g}}\). Following the same method prove an identity for \(\Delta_{\tilde{g}} \text{tr}_g\).

2) Continuing question 1, prove that
\[
\Delta_{\tilde{g}} \log \text{tr}_{\tilde{g}} g \geq -C \text{tr}_{\tilde{g}} g - C,
\]
where \(C\) depends only on \((X, \omega), F\) (this holds for any value of \(\lambda = -1, 0, 1\)). Then proceed as in class and deduce from this that
\[
\text{tr}_{\tilde{g}} g \leq Ce^{C(\varphi - \inf_X \varphi)},
\]
for a (possibly different) constant \(C\) that depends on \((X, \omega), F\).