1) Let $f$ be a holomorphic function on a domain $\Omega \subset \mathbb{C}$, and let $p > 0$. Show that $|f|^p$ is subharmonic on $\Omega$.

2) Find the Perron solution $\tilde{f}$ of the Dirichlet problem for a harmonic function on the annulus $\Omega = \{a < |z| < b\}$, $a, b > 0$, where $f : \partial \Omega \to \mathbb{R}$ is given by $f(z) = A$ for $|z| = a$ and $f(z) = B$ for $|z| = b$.

3) Let $\Omega \subset \mathbb{C}$ be a domain and $u : \Omega \to \mathbb{R}$ a harmonic function. If $u^2$ is also harmonic, show that $u$ is constant.

4) Let $\Omega \subset \mathbb{C}$ be a domain and $u : \Omega \to \mathbb{C}$ a function whose real and imaginary parts are harmonic functions. If $u^2$ is also harmonic, show that $u$ is either holomorphic or antiholomorphic.