1) Let $E \to M$ be a vector bundle, and $F \to M$ be the trivial rank $r$ vector bundle. Show that $E \otimes F \cong E \oplus \cdots \oplus E$ ($r$ times).

2) Let $M$ be a manifold. Show that the set of isomorphism classes of line bundles over $M$ is an Abelian group, with group multiplication being tensor product.

3) Let $\pi : E_1 \to \mathbb{RP}^1$ be the line bundle over $\mathbb{RP}^1$ that we considered in homework 7. Show that the total space of $E_1$, which is a 2-dimensional manifold, is not orientable.

4) Let $M$ be an $n$-manifold, and $X_1, \ldots, X_n$ be smooth vector fields on an open set $U \subset M$ which are linearly independent at each point. Let $\alpha_1, \ldots, \alpha_n$ be the smooth 1-forms on $U$ which at each point are the dual basis of these vector fields. On $U$ we can write

$$[X_i, X_j] = \sum_{k=1}^n c^k_{ij} X_k,$$

for smooth functions $c^k_{ij}$ on $U$, for all $1 \leq i, j \leq n$ (recall homework 7). Show that

$$d\alpha_k = -\sum_{1 \leq i < j \leq n} c^k_{ij} \alpha_i \wedge \alpha_j,$$

on $U$, for all $1 \leq k \leq n$. 