Solve all the problems below. Each problem is worth 10 points.

1) Let \( f : B^2 \to \mathbb{C} \) be a continuous map such that \( f(x) \neq 0 \) for all \( x \in S^1 \subset B^2 \). Define a continuous map \( g : (S^1, 1) \to (S^1, 1) \) (where we are viewing \( S^1 \subset \mathbb{C} \), so 1 is the same as the point \((1, 0) \in S^1 \subset \mathbb{R}^2\)), by

\[
g(x) = \frac{f(x)}{|f(1)|}.
\]

(a) Show that if \( g \neq 0 \) in \( \pi_1(S^1, 1) \) then \( f \) must have a zero in \( B^2 \).

(b) Find an example where \( g = 0 \) in \( \pi_1(S^1, 1) \) and \( f \) has a zero in \( B^2 \).

(c) Find an example where \( g = 0 \) in \( \pi_1(S^1, 1) \) and \( f \) does not have a zero in \( B^2 \).

2) In the same setting as problem 1, suppose \( g \neq 0 \) in \( \pi_1(S^1, 1) \) (so that \( f \) must have a zero in \( B^2 \) by problem 1(a)). Given \( h : B^2 \to \mathbb{C} \) a continuous map such that

\[|h(x)| < |f(x)|,\]

for all \( x \in S^1 \), then \( f + h \) must have a zero in \( B^2 \).

3) Is there a retraction \( r : X \to A \) where \( X = S^1 \times B^2 \) and \( A = S^1 \times S^1 \subset X \) where \( S^1 \subset B^2 \) is the usual inclusion of the circle as the boundary of the disc?

4) Is there a retraction \( r : X \to A \) where \( X = S^1 \times B^2 \) and \( A \subset X \) is the subspace pictured below, which is homeomorphic to a circle \( S^1 \)?