Problems for MATH 446-2, I

1. Represent the universal bundle $E(1, N)$ and the tangent bundle $T_{\mathbf{P}^N}$ on $\mathbf{P}^N$ by a cocycle $g_{ij}$ with respect to the cover $U_i = \{(z_0 : \ldots : z_N) | z_i \neq 0\}$.

2. Classify all real and complex vector bundles on $S^1$ and $S^2$ up to isomorphism. Where are the tangent bundles on that list?

3. We have shown in class that the Grassmannian $Gr(n, N)$ is a homogeneous space $GL(n + N)/P$ for some subgroup $P$. Find a subgroup $Z$ of $GL(n + N)/P$ such that any Schubert cell $C$ is a $Z$-orbit of an element $sP$ where $s$ is a permutation matrix.