Nadler

Goal: M exact sympl mfld

$M \supset \Lambda$ Lagrangian scheleton

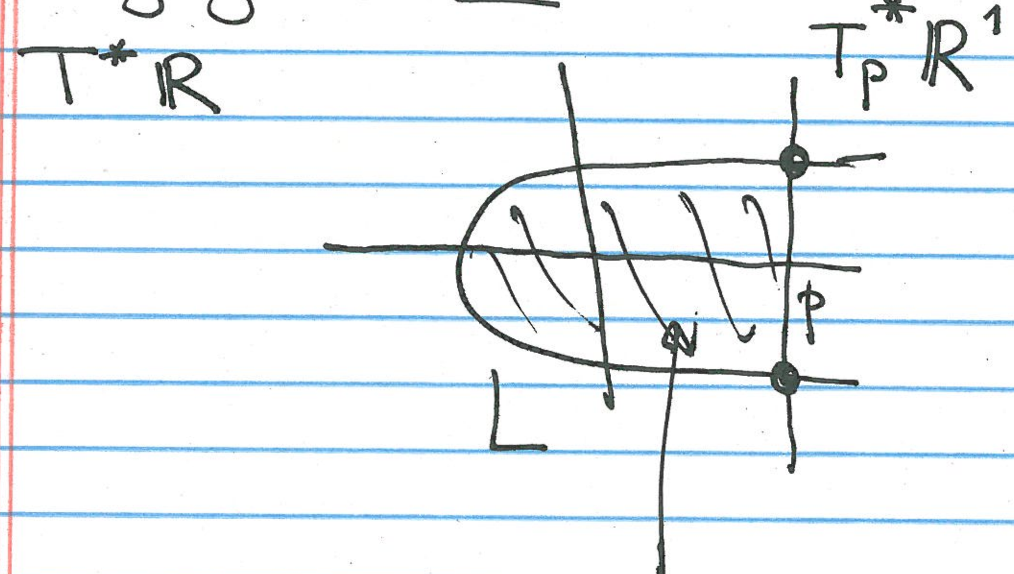
$\rightsquigarrow$ **Analysis**: Fukaya category (elliptic PDE...)

Algebra: Modules over deformation quantization

Topology: Lagrangian cobordisms

Roughly: 3 ways to assign a category. All should be the same.

Example goal: Show that the Fukaya category is **local**.



Holomorphic disc **cancels** the two intersection pts

Nevertheless: we want to look at them <u>locally</u>.

<u>Setup</u>: M exact symplectic $\supset \Lambda$ Lagrangian skeleton

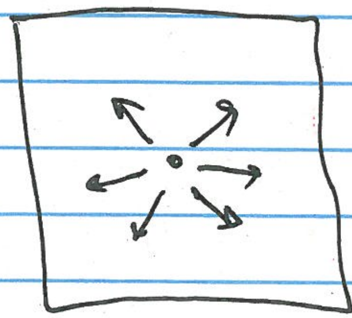
$(M, \theta, \omega = d\theta)$ — θ: 1-form; ω: symplectic

too rough... want this to be Weinstein

Liouville vector field v_θ

$h: M \to \mathbb{R}$ Morse with v_θ gradient-like

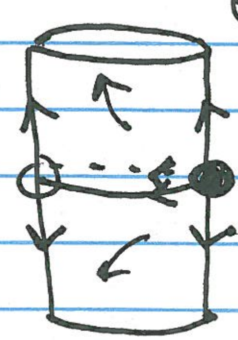
<u>Ex</u> 1) $M \simeq \mathbb{C}^n$; v_θ single zero at origin



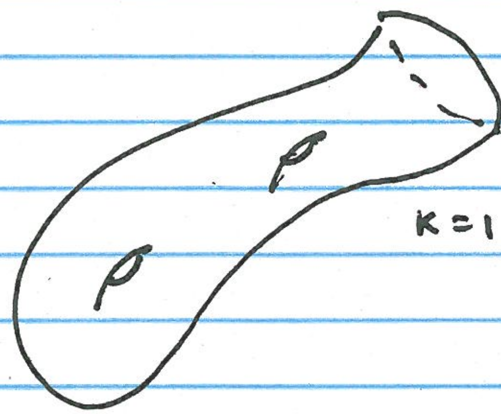
Weinstein cell

2) $M = T^*X$ $\quad v_\theta =$ dilation + grad(Morse fn)

$X = S^1$



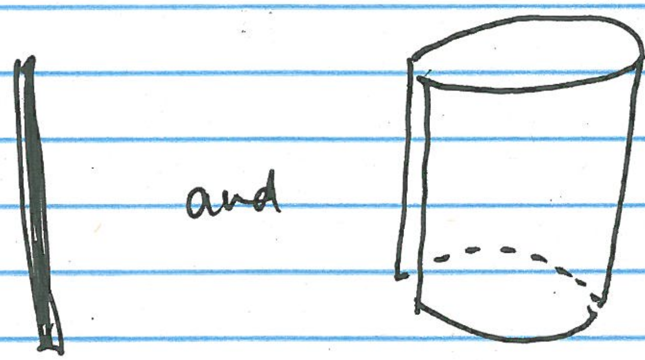
3) $M = \mathrm{Sym}^k$(punctured Riemann surf.)



Cell decomposition

$$M = \coprod_p C_p \leftarrow \text{coisotropic cells}$$

ex.:



everything that flows into a fixed points of the Liouville flow

Similarly: $K = \coprod_p c_p$ – isotropic cells homot. equ.

$$M = \coprod_p C_p \quad \text{coisotropic cells}$$
$$\cup$$
$$K = \coprod_p c_p \quad \text{isotropic cells}$$

$$E = M \setminus K \quad \underline{\text{ether}}$$
core

$$E/\mathbb{R}_+ \simeq \partial M$$

(the ether goes to ∞ by the flow).

<u>Principle</u>: A homological invariant of M = Gluing of homological invariants of these cells.

$$M \hookleftarrow C_p \twoheadrightarrow M_p$$

hamiltonian reduction of a coisotropic cell

So, what we do is a version of Morse homology.

What is Fukaya category $\mathcal{F}(M)$?

<u>Objects</u>: $L \subset M$ not necessarily compact

Should go off to ∞ in some nice regular way

+ brane structures

→ (grading; spin structure; ...)

<u>discrete</u> amount of data

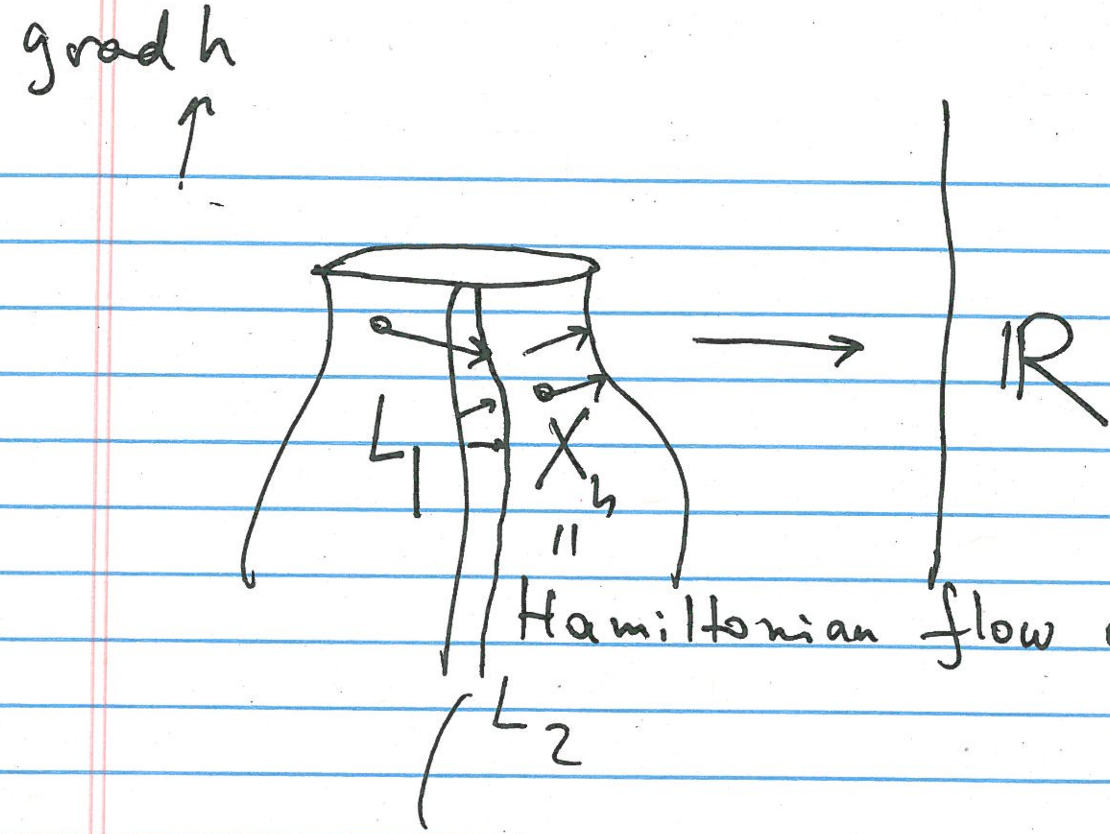
<u>Morphisms</u>: $\mathbb{C}\{L_1 \pitchfork L_2\}$ linear span

(+ gradings)

<u>Transversality</u>: on a compact set, clear how to perturb to make transversal.

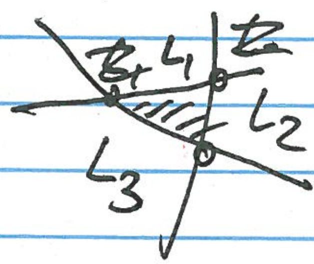
What if the intersection happens at ∞?

<u>RULE</u>: morphisms propagate forward <u>in time</u> (with the Hamiltonian flow of h)



perturb L_1 forwards in time.
That is the rule for perturbation

<u>A_∞ structure</u>: counts J-holomorphic discs



<u>Stabilization</u>

$\downarrow$

"Perfect envelope"

$F(M)$ (small) stable ∞-category

<u>Not</u> the only game in town.

By mirror symmetry: $\mathcal{F}(M) \simeq \mathrm{Perf}(?)$

and there is (?) something $\simeq$ Coh(?)

Some category bigger than $\mathcal{F}(M)$;
wrapped Fukaya category

Ex $M = T^*X \rightsquigarrow \mathcal{F}(M) \simeq Sh_c(X)$

Main Technical Thm $\quad M = \coprod_p C_p$

coisotropic cells

$\mathcal{S} = \{C_p\}_p$ stratification

Fully faithful embedding

(I) $Sh_{\mathcal{S}}(M) \longrightarrow End(\mathcal{F}(M))$

$\mathcal{S}$-constructible

Monoidal structure $= \otimes$

Dévissage pattern: $M^\circ \subset M$

open union of cells

$N \twoheadleftarrow C \subset M$ closed union of cells

(II) $\mathcal{F}(N) \rightleftarrows \mathcal{F}(M) \rightleftarrows \mathcal{F}(M^\circ)$

point: • (II) determined by (I)...

• for $M = T^*X$ as above, get standard dévissage pattern for constructible sheaves...

• everything is a stable ∞ category, i.e. has cones etc., and ~~everything~~ is preserved by ~~this structure~~ these functors...

(micro, if $M = T^*X$) $\equiv$

Localization $\quad L \in \mathcal{F}(M)$

1) null locus $\quad n(L) =$ conical

$\exists$ conic $U \subset M$

$s(L) = M - n(L)$

sing supp

$\hom_{F(M)}(L, p) \simeq 0 \quad \forall p \subset U$

Def $\Lambda \subset M$ conical Lagr

assume $K \subset \Lambda$

$\mathcal{F}_\Lambda(M) \subset \mathcal{F}(M)$ full subcat

$ss(L) \subset \Lambda$

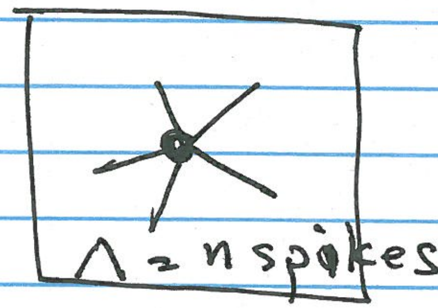
Thm There exists a sheaf of (small) stable ∞-cats $\mathcal{F}_\Lambda$ on ~~same~~ conic topology of M s.t.

1) $\mathrm{Supp}(\mathcal{F}_\Lambda) \simeq \Lambda$
2) $\Gamma(M, \mathcal{F}_\Lambda) \simeq \mathcal{F}_\Lambda(M)$
3) $\mathcal{F}_\Lambda|_{M^0} \simeq \mathcal{F}_{\Lambda^0}$ $\quad \Lambda^0 = \Lambda \cap M^0$

<u>The way to check it</u>: by dévissage pattern.

Ex $M = T^*X \rightsquigarrow$ Microlocal constructible sheaves

$M = \mathbb{C}$



$\mathcal{F}_\Lambda(M) = \mathrm{Mod}(A_{n-1}\text{-quiver})$

(Restrictions to spikes) =

= Vector spaces sitting at nodes of