Instructions: Write your name and I.D. number above. Show all work on these pages, and make sure that your final answer is clearly shown. No books and calculators are permitted. Check that this exam contains pages 1–10. Good Luck, and have a great Spring break!

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1. (20 pts) Use Green’s Theorem to evaluate

\[ \int_C (x + 2y)dx + y^4dy, \]

where \( C \) is a circle \( x^2 + y^2 = 4. \)

Answer: \(-8\pi \).
2. (20 pts) Evaluate the integral
\[ \int_C x \cos z \, ds, \]
where \( C \) is the circular helix given by equations \( x = \cos t, \quad y = \sin t, \quad z = t, \quad 0 \leq t \leq 2\pi. \)
Answer: \( \sqrt{2\pi}. \)
3. (20 pts) Calculate the following iterated integral by reversing the order of integration:

\[
\int_0^1 \int_{x/3}^1 \frac{4}{y^4 + 1} \, dy \, dx.
\]

Answer: \( \ln 2 \).
4. (20 pts) Find the volume of the region bounded by two paraboloids $z = x^2 + y^2 - 2$ and $z = 6 - x^2 - y^2$.

Answer: $16\pi$. 
5. (20 pts) a) Show that $\mathbf{F}(x, y) = y \sin x \mathbf{i} + (e^y - \cos x) \mathbf{j}$ is a conservative vector field and find a function $f(x, y)$ such that $\mathbf{F} = \nabla f$.

   Answer: $f = -y \cos x + e^y$.

b) Let $C$ be the arc of the ellipse $(x - \pi)^2 + \pi^2 y^2 = \pi^2$ from $(0, 0)$ to $(\pi, 1)$. Evaluate

$$\int_C y \sin x \, dx + (e^y - \cos x) \, dy.$$ 

Answer: $e$. 
6. (20 pts) Evaluate the double integral
\[
\int \int_R (y - x) \, dA,
\]
where \( R \) is bounded by \( y = x - 1, y = x + 4, y = 1 - 2x, \) and \( y = -2x. \)
Answer: \( \frac{5}{2}. \)
7. (20 pts) (a) Show that the vector field \( \mathbf{F} = 2xycosz \, \mathbf{i} + (x^2cosz - 2yz) \, \mathbf{j} + (-x^2ysinz - y^2) \, \mathbf{k} \) is irrotational.
(b) Find a potential function for \( \mathbf{F} \).

Answer: \( x^2ycosz - y^2z \).
8. (20 pts) (a) Find the outward flux of the vector field

\[ \mathbf{F} = (x^3 + 2xe^y \arctan y)i + (y^3 - \tan(x + z))j + (z^3 - 2ze^y \arctan y)k \]

across the boundary of the unit ball.

Answer: \( \frac{12}{5}\pi \).

(b) Find the flux of \( \mathbf{F} = -x \mathbf{i} - y \mathbf{j} + 2 \mathbf{k} \) across the paraboloid \( z = 4 - (x^2 + y^2) \) for \( z \in (0, 4) \). Note that the bottom is not included.

Answer: \(-8\pi \).
9. (20 pts) Use Stokes’ Theorem to evaluate the integral of $\mathbf{F} = y \mathbf{i} + z^2 \mathbf{j} + x^2 \mathbf{k}$ along the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$, oriented counterclockwise with respect to the positive direction of $\mathbf{k}$.

Answer: $-\pi$. 
10. (20 pts) Compute the integral

\[ \int \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS, \]

where \( \mathbf{F} = -y \mathbf{i} + x \mathbf{j} + xyz^5 \mathbf{k} \), and \( S \) is the portion of the cone \( z = r \) between the plane \( z = 0 \) and \( z = 3 \).

Answer: \( 18\pi \).