1. Using the Principle of Extension, prove the following (\( \triangle \) is symmetric difference):
   (a) \( A \triangle (B \triangle C) = (A \triangle B) \triangle C \).
   (b) \( A \cap (B \triangle C) = (A \cap B) \triangle (A \cap C) \).
   You may use properties stated in the Notes or proven in previous homework assignments.

2. Prove that the following statements are equivalent:
   (a) \( A \subseteq B \).
   (b) \( A \cap B = A \).
   (c) \( A \cup B = B \).
   (d) \( A - B = \emptyset \).
   (One way to prove the equivalence is to prove the chain of implications: (a) \( \Rightarrow \) (b) \( \Rightarrow \) (c) \( \Rightarrow \) (d) \( \Rightarrow \) (a).)

3. Prove the following:
   (a) \( A \triangle B = \emptyset \iff A = B \).
   (b) \( A \triangle B \subseteq S \iff A - S = B - S \).

4. For \( n \geq 1 \) let \( A_n \) be the following interval: \( A_n = \left[ -2, (-1)^n(1 + 1/n) \right] \). Find the following \((k \in \mathbb{Z}^+)\):
   (a) \( \bigcap_{n=k}^{\infty} A_n \),
   (b) \( \bigcup_{n=k}^{\infty} A_n \),
   (c) \( \bigcup_{k=1}^{\infty} \left( \bigcap_{n=k}^{\infty} A_n \right) \),
   (d) \( \bigcap_{k=1}^{\infty} \left( \bigcup_{n=k}^{\infty} A_n \right) \).

5. Find the properties (reflexive, transitive, symmetric, antisymmetric) verified by the following relations:
   (a) Strict inequality of integers: \( x \mathcal{R} y \iff x < y \).
   (b) Set disjointness: \( A \mathcal{R} B \iff A \cap B = \emptyset \).
   (c) The following relation on \( \mathbb{Q} \): \( x \mathcal{R} y \iff x - y \in \mathbb{Z} \).
   (d) The following relation on \( \mathbb{Q} \): \( x \mathcal{R} y \iff x - y \in \mathbb{N} \).

6. We define the following relation on \( \mathbb{N} \): \( x \preceq y \) if and only if
   (a) \( x \) is even and \( y \) is odd, or
   (b) \( x \) and \( y \) have the same parity and \( x \leq y \).
   For instance: \( 2 \preceq 6, 3 \preceq 7, 2 \preceq 5, 24 \preceq 3 \).
   1. Prove that \( \preceq \) is a total order.
   2. For each of the following numbers find a successor an immediate successor, a predecessor and an immediate predecessor, or show that there is none: 3, 2, 1, 0.

7. Prove that the following is an equivalence relation on \( \mathbb{N}^2 \):
   \( (a, b) \mathcal{R} (a', b') \iff a + b' = a' + b \).
8. Let $U$ be a nonempty set, and let $\mathcal{P}(U)$ be its power set. Let $S \in \mathcal{P}(U)$ be a subset of $U$. In $\mathcal{P}(U)$ we define the following relation: $A \mathcal{R} B \iff A \triangle B \subseteq S$.
(a) Prove that $\mathcal{R}$ is an equivalence relation.
(b) Prove that all equivalence classes have the form $C_A = \{A \cup S' \mid S' \in \mathcal{P}(S)\}$ for $A \in \mathcal{P}(S)$.

9. Find (if they exist) the greatest element, the least element, the least upper bound and the greatest lower bound for each of the following subsets of $(\mathbb{R}, \leq)$:
(a) $A = \{(-1)^n + 1/n \mid n \in \mathbb{Z}^+\}$.
(b) $B = \{x \in \mathbb{R} \mid x^2 < 5\}$.
(c) $C = \{x \in \mathbb{Q} \mid x^2 < 5\}$.
(d) $D = \{x \in \mathbb{Z} \mid x^2 < 5\}$.

10. Draw the Hasse diagram for the poset $P = (\{2, 3, 4, 5, 6, 7, 8, 9, 10\}, |)$, where “|” represents divisibility. Find the minimal and maximal elements in $P$. 