1. Write the truth table for the following statements.
   (a) \( p \rightarrow (q \rightarrow r) \)
   (b) \( (p \rightarrow q) \rightarrow r \)

2. Let \( p \) and \( q \) be primitive statements such that \( p \rightarrow q \) is false. Find the truth value of the following:
   (a) \( q \rightarrow p \)
   (b) \( \neg p \rightarrow \neg q \)
   (c) \( \neg q \rightarrow \neg p \)

3. Write the following statement in \textit{conjunctive normal form}:
   \( p \leftrightarrow (q \rightarrow r) \)

4. We define the connective \textit{nand} by:
   \( p \uparrow q \iff \neg(p \land q) \)

   Make its truth table. Write the following statements using \( \uparrow \) only
   (a) \( \neg p \)
   (b) \( p \land q \)
   (c) \( p \lor q \)
   (d) \( p \rightarrow q \)
   (e) \( p \leftrightarrow q \)

   (For instance: \( \neg p \iff p \uparrow p \).)

5. Use truth tables to determine if the following logical equivalences are correct (in exercise 1 you already made the truth table for the left hand sides):
   (a) \( p \rightarrow (q \rightarrow r) \iff (p \land q) \rightarrow r \)
   (b) \( (p \rightarrow q) \rightarrow r \iff (\neg p \rightarrow r) \land (q \rightarrow r) \)


7. Check the following logical implications:
   (a) \( \neg p \rightarrow p \Rightarrow p \)
   (b) \( p \Rightarrow (\neg p \rightarrow q) \)
   (c) \( (p \rightarrow q) \Rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow r)) \)
8. Given the following set of premises:
   1. \((\neg p \rightarrow p) \rightarrow p\)
   2. \(p \rightarrow (\neg p \rightarrow q)\)
   3. \((p \rightarrow q) \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow r))\)
   and using only Modus Ponens as rule of inference:

\[
\begin{align*}
   & p \\
   & p \rightarrow q \\
   \hline
   & \therefore q
\end{align*}
\]

prove: \(p \rightarrow p\).

9. What truth values of \(p\), \(q\) and \(r\) are compatible with the following set of premises?
   1. \(p \leftrightarrow \neg q\)
   2. \(q \leftrightarrow \neg r\)
   3. \(r \leftrightarrow (\neg p \land \neg q)\)

10. Consider the following statements:
    (a) \(\forall x \forall y (x \leq y)\)
    (b) \(\forall x \exists y (x \leq y)\)
    (c) \(\exists x \forall y (x \leq y)\)
    (d) \(\exists x \exists y (x \leq y)\)
    Determine their truth value assuming that the universe of discourse is:
        (1) The set of all integers.
        (2) The set of positive integers.
        (3) The set of negative integers.
        (4) The set \(A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}\).

11. Assume that the universe of discourse is the set of integers. Prove the following, stating
    the method or principle being used:
    (a) \(\exists x (x^2 = x)\).
    (b) \(\forall x \forall y (x + y < 10 \rightarrow x < 2 \lor y < 8)\).

12. Write the negation of the following quantified statement in prenex normal form, leaving
    the statement inside in disjunctive normal form:
    \(\forall \varepsilon [\varepsilon > 0 \rightarrow \exists N \forall n (n > N \rightarrow |a_n| < \varepsilon)]\).