These problems are for your own benefit, please do not submit solutions for grading.

1) Let

\[ \varphi(u, v) = (u, v, u^2 - v^2), \quad \psi(u, v) = \left( \frac{u + v}{2}, \frac{u - v}{2}, uv \right), \]

where \((u, v) \in \mathbb{R}^2\). Check that these are two parametrizations of the same surface \(S\). What is \(S\)? Then consider the smooth map \(F : \mathbb{R}^3 \to \mathbb{R}^3\) given by

\[ F(x, y, z) = (2x, 2y, 4z). \]

Show that \(F\) maps \(S\) to \(S\), i.e. \(F\) defines a smooth map \(F : S \to S\). Lastly, let \(\tilde{F} = \psi^{-1} \circ F \circ \varphi\) and compute the Jacobian matrix of \(\tilde{F}\), i.e. compute the expression of \(dF\) with respect to the canonical bases given by these two parametrizations.

2) Does the equation \(z^2 = x^2 \cos y + 1\) define an orientable surface?

3) Let \(\gamma(t)\) be a smooth curve in \(\mathbb{R}^3\) parametrized with respect to arclength and with curvature never zero, and define

\[ \varphi(u, v) = \gamma(u) + vB(u), \]

where \(B(t)\) is the binormal vector to \(\gamma(t)\). This defines a surface (you don’t need to check it). Calculate the first fundamental form of the resulting surface.