440-1,2,3 GEOMETRY/TOPOLOGY

Fall (Introduction to differentiable manifolds): Differentiable manifolds; implicit function theorem and Sard's theorem;tangent vectors and tensors; vector fields and flows; integral manifolds and Frobenius's theorem; differential forms, orientation and integration; Riemannian metrics, the Levi-Civita connection, geodesics, and the exponential map.

Winter (Introduction to algebraic topology): The fundamental group of a space; covering spaces; the Van-Kampen theorem; singular homology; Künneth formula, homotopy invariance, excision; homological algebra; degree and CW homology.

Spring (Cohomology): Singular cohomology; the cup product; de Rham cohomology; sheaf cohomology; the de Rham theorem; orientability and Poincaré duality.

440-1. FALL: Introduction to differentiable manifolds

Text: *Geometry of Manifolds* by Richard L. Bishop and Richard J. Crittenden, and *Topology and Geometry* by Glen E. Bredon

(1) Differentiable manifolds: definition and examples (including projective spaces); submersion, immersion and embedding (submanifolds).

(2) Implicit function theorem; critical values and Sard's theorem.

(3) Tangent vectors as derivations; tangent space; basis in local coordinates; tangent vector fields and their flows.

(4) Lie brackets and their geometric meaning; distributions and integral manifolds; Frobenius's theorem.

(5) Cotangent vectors; tensors and contractions; differential forms and exterior differentiation; Lie derivatives and Cartan's homotopy formula.

(6) Orientability and integration of differential forms on manifolds; Stokes's theorem; examples.

(7) Riemannian manifolds; arc length and Riemannian distance; volume form and measure.

(8) The Levi-Civita connection; Christoffel symbols; covariant differentiation of tensor fields; parallel transport; examples.

(9) Variation of arclength; geodesics; exponential map; Gauss's lemma and local normal coordinates.

(10) (Other optional topics, if time allows, not included in the prelim exam) Riemannian curvature tensor; definition of Lie group and simple examples; Ehresmann's fibration theorem.

440-2. WINTER: Introduction to algebraic topology

Text: *Algebraic Topology* by Allen Hatcher.

(1) The fundamental group; differential versus continuous fundamental group.

(2) Covering spaces; fundamental theorem of algebra.

(3) The universal cover and Galois theory of covering spaces, generalities on pushouts of group theory.

(4) Van Kampen’s theorem after Grothendieck, higher-dimensional spheres are simply-connected.

(5) Examples. Riemann surfaces, projective spaces, construction of K(G,1)-spaces.

(6) Singular homology and the Eilenberg-Zilber theorem.

(7) The Künneth theorem; homotopy invariance of homology; excision.

(8) Homological algebra including the universal coefficient theorem.

(9) Degree and CW homology; Euler characteristics.

440-3 SPRING: Cohomology

Text: *Algebraic Topology* by Allen Hatcher and *Foundations of Differentiable Manifolds and Lie Groups* by Frank Warner.

(1) Singular cohomology, formal properties that follow from homology.

(2) The cup product and the cap product, graded-commutativity.

(3) de Rham cohomology, connection to div-grad-curl.

(4) Sheaf cohomology and Čech cohomology.

(5) The de Rham theorem, the Poincaré lemma.

(6) Orientability and Poincaré duality.

(7) Cohomology with compact support, Borel-Moore homology, other forms of Poincaré duality.

(8) (Other optional topics, if time allows, not included in the prelim exam.) Local systems and monodromy. Differential equations and the Riemann-Hilbert correspondence.