10. (a) Define *geodesic*. (b) Consider the surface M parametrized by $\Phi(s,\theta) = (r(s)\cos\theta, r(s)\sin\theta, z(s))$, where $\theta \in (0,\pi)$ and $s \in (0,1)$ is the arc length of the curve $(r(s), z(s))$ in the (r,z) plane (i.e. $r'(s)^2 + z'(s)^2 = 1$). Assume $r(s) > 0$. Prove that if $\theta_0 \in (0,\pi)$ then the curve $f(s) = \Phi(s,\theta_0)$ is a geodesic.

11. (a) State the definition of a Jacobi field; (b) Let M be a two dimensional Riemannian manifold which is diffeomorphic to $\mathbb{R}^2$. The Riemannian metric in polar coordinates is given by

$$ds^2 = dr^2 + (\cosh r)^2 d\theta^2.$$

Use Jacobi fields to show that M has constant curvature -1.

12. (a) State the Hopf-Rinow theorem concerning the relaltion between the completeness and the exponential maps of a Riemannian manifold; (b) Suppose that M is a complete Riemannian manifold. Show that it is compact if and only if the diameter of M is finite.