

**MATH 520-1**  
**WINTER 2021**

**Topics in Mathematical Physics**

Instructor : Tuca Auffinger  
Office : Living Room  
Email : tuca@math.northwestern.edu  
Office Hours : 24/7 by Zoom (just send me an email).

This is a topics class that touches a few classical and recent areas of probability theory.

I plan to cover six independent topics. They are listed below. Classes will proceed through the following routine: (a) Introduce a new problem/topic, cover what is known/not known. (b) Explain one fundamental tool related to that area in detail. (c) If not last week of classes, go back to (a) and repeat. If last week of classes, end course.

The schedule is flexible and I guess each area may take 3 to 5 lectures.

Topics:

- (1) KPZ universality
  - (a) Particle systems, totally asymmetric simple exclusion process, RSK correspondence.
  - (b) Young tableaux: limit shapes and fluctuations.
- (2) Random metrics on  $\mathbb{Z}^d$ .
  - (a) Subadditive Ergodic Theorem and Gromov-Hausdorff convergence.
  - (b) Asymptotic direction of geodesics.
- (3) Brownian motion and PDE's.
  - (a) Dirichlet's problem, Feynman-Kac formula.
  - (b) Polar sets for Brownian motion, capacity, and existence of bounded harmonic functions on domains.
- (4) Random Graphs
  - (a) Phase transitions for Erdos-Renyi.
  - (b) Preferential attachment models.
- (5) Riemann Zeta Function and Log Correlated Fields
  - (a) Branching Brownian motion, Gaussian Free Field, extremal processes.
  - (b) Fyodorov-Hiary-Keating conjecture.
- (6) The de Bruijn-Newman constant.
  - (a) Relation to Lee-Yang's Theorem.
  - (b) Rodgers-Tao's inequality.

There are positive and negative aspects in this plan. First, we will not be able to go deep in any of these areas. Each item could (and deserves to) be developed as a single course. I will try to mitigate this problem by providing enough references and a possible study plan for each one. So if you fall in love with a topic, you will get guidance and tons of stuff to read. At the same time, if you are not a fan of a particular subject, you can always turn off your camera and wait for the next one to see something new.

Think this class as a tasting menu. Each dish will be small but unique with tons of flavor. They will also fit a broad narrative. Will this experience help you to become a better chef? Probably not. It should at least bring perspective and hopefully some things to think about.

**Pre-requisites:** Math maturity should suffice. 450-1,2,3 (probability) is desirable so is 410-2 (functional).

**Classes:** Classes are online, synchronous. They will be recorded and posted on Canvas.

**References:** Tons. They will be shared in class as we go.

**Assessment :** There will be no exams. Assessment will be based on 6 problem sets - one for each topic. If you want credit for this class you will need to submit all assignments.

**Another important info:** Any student with a disability requesting accommodations is required to register with Services for Students with Disabilities (ssd@northwestern.edu; 847-467-5530) and provide an accommodation notification from SSD to his/her professor, preferably within the first two weeks of class. All information will remain confidential.

### *Terms of Service:*

This class will be recorded by the instructor for educational purposes. These recordings will be shared only with students enrolled in the course and will be deleted at the end of the Winter Quarter 2021. Your instructor will communicate how you can access the recordings.

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