NAME: ______________________________

WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION

Problem A1. Find the integer values of x for which the following function takes integer values:

$$f(x) = \frac{x^2}{x+3}\,.$$

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Problem A2. On a table there are 100 tokens. Taking turns two players remove 5, 6, 7, 8, 9 or 10 tokens, at their choice. The player that removes the last token wins. Find a winning strategy and determine which player will be the winner.

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Problem A3. In a group of n people ($n \geq 2$) each person picks another person at random and, at the sound of "now!", throws a pie to him/her. Assume that all pies have the same probability p of hitting their target, and if the pie misses its intended target it does not hit anybody else. What is the expected number of people not hit by a pie?

NORTHWESTERN UNIVERSITY
SELECTION TEST

Monday, October 13, 2003

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Problem A4. If $x \neq 0$ prove that $\dfrac{\sin x}{x} = \displaystyle\prod_{n=1}^{\infty} \cos\left(\frac{x}{2^n}\right)$.

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Problem A5. In the figure OP is the bisector of angle ROS. Prove than $1/|OR|+1/|OS| = 1/|OR'| + 1/|OS'|$.

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Problem A6. We have a calculator with two registers R_1 and R_2, and four operations:

(1) $R_1 + R_2 \to R_2$ (add the content of register R_1 to register R_2.)
(2) $-R_1 + R_2 \to R_2$ (subtract the content of register R_1 from register R_2.)
(3) $R_1 + R_2 \to R_1$ (add the content of register R_2 to register R_1.)
(4) $R_1 - R_2 \to R_1$ (subtract the content of register R_2 from register R_1.)

For instance, if $R_1 = x$ (register R_1 contains the number x) and $R_2 = y$ (R_2 contains y), after applying the operation $R_1 + R_2 \to R_2$ we end up with $R_1 = x$ and $R_2 = x + y$. Assume that initially we have $R_1 = x$ and $R_2 = y$, where x and y are arbitrary numbers. For each of the following tasks describe a sequence of operations that would allow us to perform the task, or prove that it is impossible:

(1) Swap the contents of registers R_1 and R_2 changing the sign of y in the process, so we would end up with $R_1 = -y$, $R_2 = x$.
(2) Swap the contents of registers R_1 and R_2, so that we would end up with $R_1 = y$, $R_2 = x$.