NAME: ______________________________

2006 NU PUTNAM TEAM SPRING COMPETITION

Problem A1. Find the sum of the digits of

$$9 + 99 + 999 + \cdots + \overbrace{99\ldots9}^{99}\,.$$

NAME: ______________________________

2006 NU PUTNAM TEAM SPRING COMPETITION

Problem A2. Let a, b two positive real numbers. Prove that $a^a + b^b \geq a^b + b^a$.

NAME: ____________________

2006 NU PUTNAM TEAM SPRING COMPETITION

Problem A3. Let x, y, z positive real numbers. Find the minimum value of the function

$$f(x, y, z) = \left(x + \frac{1}{y}\right)\left(y + \frac{1}{z}\right)\left(z + \frac{1}{x}\right).$$

NAME: ____

2006 NU PUTNAM TEAM SPRING COMPETITION

Problem A4. Let S be a set of real numbers which is closed under multiplication (that is, if a and b are in S, then so is ab). Let T and U be disjoint subsets of S whose union is S. Given that the product of any *three* (not necessarily distinct) elements of T is in T and that the product of any three elements of U is in U, show that at least one of the two subsets T, U is closed under multiplication.

NAME: ________________________________

2006 NU PUTNAM TEAM SPRING COMPETITION

Problem A5. Prove that if we select $n + 1$ numbers from the set $S = \{1, 2, 3, \ldots, 2n\}$, among the numbers selected there are two such that one is a multiple of the other one.

NAME: ____

2006 NU PUTNAM TEAM SPRING COMPETITION

Problem A6. The numbers $1, 2, \ldots, 17$ are placed inside the 17 circles of the figure below, so that the sum along all sides and along all diagonals is the same (note that there are eight sides and eight diagonals.) Prove that there is only one number that can be placed in the center, and find it.